Longitudinal and Lateral Control of Truck Platoons Based on Finite-time Sliding Mode *

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***Abstract*—Longitudinal and lateral control of truck platoons are considered in this paper, in which a distributed controller is designed. In which the characteristics of truck with nonlinearity and tire force coupling are considered, a five-degree-of-freedom dynamics model of truck and tire model of "Magic Formula" are introduced. Simultaneously, a longitudinal model to ensure string stability and a lateral lane-keeping model are developed in this paper. On this basis, it is developed that a longitudinal and lateral decoupling controller based on finite-time convergence. Co-simulation experiments are provided on joint simulation platform of Trucksim/Simulink, which the simulation results show that the controller can achieve fast convergence of longitudinal and lateral errors and string stability of platoon.**

***Index Terms*—Truck platoon, longitudinal and lateral motion decoupling, sliding mode control, finite-time stability, string stability**

I. Introduction

Control of vehicle platoon originates from the study of collaboration problems of multi-intelligent agent systems [1]. Truck platoon has advantages of saving fuel consumption, reducing emissions, eliminating truck collision risk, and improving road capacity. Truck is a nonlinear system due to strong coupling of lateral and longitudinal dynamics, tire nonlinearity and parametric uncertainty. Thus, it has a profound research significance to design longitudinal and lateral controllers for truck platoons.

The objective of longitudinal control of truck platoon is to track the desired speed or the desired spacing, and the objective of lateral control is to ensure that the truck does not beyond the lane boundary [2]. The sliding mode control (SMC) technique has the advantage of fast convergence and being insensitive to external disturbances, and it is widely used in the control of truck platoon.

II. Problem Formulation

Considering a homogeneous truck platoon consisting of a lead truck and N following trucks, suppose that the lead truck is uncontrolled and drives along the desired path, and the motion of the truck platoon is decoupled into a longitudinal motion along the desired path and a lateral motion of lane-keeping.

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A. Five-Degree-of-Freedom Dynamics Model of Truck

The five-degree-of-freedom dynamics model of truck [3] can be expressed as follows

$$\begin{cases} \dot{v}_x = \dfrac{1}{m}\left(F_{xf}\cos\delta_f - F_{yf}\sin\delta_f + F_{xr}\right) + v_y\omega \\ \dot{v}_y = \dfrac{1}{m}\left(F_{xf}\sin\delta_f + F_{yf}\cos\delta_f + F_{yr}\right) - v_x\omega \\ \dot{\omega} = \dfrac{1}{I_z}\left(aF_{xf}\sin\delta_f + aF_{yf}\cos\delta_f - bF_{yr}\right) \\ \dot{\omega}_f = \dfrac{1}{J_f}\left(T_{df} - RF_{xf}\right) \\ \dot{\omega}_r = \dfrac{1}{Jr}\left(T_{dr} - RF_{xr}\right) \end{cases} \tag{1}$$

where v_x, v_y and ω are the longitudinal velocity, lateral velocity, yaw angle rate of truck. ω_f and ω_r are the angular velocity of front and rear wheels; m is the total mass of truck; I_z is the moment of inertia around yaw axis; a and b are the distances from front axle and rear axle to mass center; J_f and J_r are the moment of inertia of front and rear wheel; δ_f is the lateral control input representing steering angle of the front wheel; T_{df} and T_{dr} are the longitudinal control input representing the torque of front and rear wheel; F_{xf}, F_{xr}, F_{yf} and F_{yr} are the longitudinal forces of front wheel, longitudinal forces of rear wheel, lateral forces of front wheel and lateral forces of front wheel, respectively; R is the wheel rolling radius.

B. "Magic Formula" of Tire Model

The "Magic Formula" nonlinear tire model is expressed as follows

$$\begin{cases} F_{yf} = D\sin\left(C\arctan\left(B\alpha_f - E\left(B\alpha_f - \arctan B\alpha_f\right)\right)\right) \\ F_{yr} = D\sin\left(C\arctan\left(B\alpha_r - E\left(B\alpha_r - \arctan B\alpha_r\right)\right)\right) \end{cases} \tag{2}$$

where α_f and α_r are the slip angles of front and rear tires; B, C, D and E are the parameters of tires. The slip angles of front and rear tires can be calculated as follows

$$\begin{cases} \alpha_f = \delta_f - \arctan(\dfrac{v_y + a\omega}{v_x}) \\ \alpha_r = -\arctan(\dfrac{v_y - b\omega}{v_x}) \end{cases} \tag{3}$$

C. Longitudinal Error Model of Truck Platoon

The longitudinal position and longitudinal velocity of lead truck are given as $x_0(t)$ and $v_{x,0}(t)$. For each following truck i, $i \in 1,\ 2 \ldots N$, describe its longitudinal position and longitudinal velocity as $x_i(t)$ and $v_{x,i}(t)$. Define L as the desired spacing, and the integrated spacing error of the ith following truck in the platoon as follows

$$e_{x,i}(t) = \sigma_1(x_i(t) - (x_0(t) - iL)) + \sigma_2(x_i(t) - (x_{i-1}(t) - L)) \tag{4}$$

where σ_1 and σ_2 are positive constant parameters, and $\sigma_1 + \sigma_2 = 1$.

Calculating the derivative of (4), we have

$$\begin{cases} \dot{e}_{x,i}(t) = v_{x,i}(t) - \sigma_1 v_{x,0}(t) - \sigma_2 v_{x,i-1}(t) \\ \ddot{e}_{x,i}(t) = a_{x,i}(t) - \sigma_1 a_{x,0}(t) - \sigma_2 a_{x,i-1}(t) \end{cases} \tag{5}$$

where $a_{x,i}(t)$ and $a_{x,0}(t)$ are longitudinal acceleration of following truck i and lead truck.

D. Lane-Keeping Model of Truck Platoon

The integrated lane-keeping error of following truck i in the platoon as follows

$$e_{y,i}(t) = \sigma_3 y_i(t) + \sigma_4 \left(\varphi_i(t) - \varphi_{d,i}(t)\right) \tag{6}$$

where $y_i(t)$ is the lateral position error and lane centerline of following truck i; $\varphi_i(t)$ and $\varphi_{d,i}(t)$ are actual yaw angle and desired yaw angles; σ_3 and σ_4 are positive constant parameters.

Calculating the derivative of (6),we have

$$\begin{cases} \dot{e}_{y,i}(t) = \sigma_3 v_{y,i}(t) + \sigma_4(\omega_i(t) - \omega_{d,i}(t)) \\ \ddot{e}_{y,i}(t) = \sigma_3 a_{y,i}(t) + \sigma_4(\dot{\omega}_i(t) - \dot{\omega}_{d,i}(t)) \end{cases} \tag{7}$$

where $v_{y,i}(t)$ and $a_{y,i}(t)$ are the lateral velocity and lateral acceleration of following truck i; $\omega_i(t)$, $\dot{\omega}_i(t)$, $\omega_{d,i}(t)$ and $\dot{\omega}_{d,i}(t)$ are the yaw angle rate, yaw angle acceleration, desired yaw angle rate and desired yaw angle acceleration.

E. Control Objective of Truck Platoon

1) Longitudinal finite-time stability:

$$\begin{cases} \lim\limits_{t \to t_{x,i}} |e_{x,i}(t)| = 0 \\ \lim\limits_{t \to t_{x,i}} |e_{v,i}(t)| = 0 \end{cases} \tag{8}$$

2) Lateral finite-time stability:

$$\begin{cases} \lim\limits_{t \to t_{y,i}} |y_i(t)| \leq 0.5 \\ \lim\limits_{t \to t_{y,i}} |\omega_i(t) - \omega_{d,i}(t)| = 0 \end{cases} \tag{9}$$

III. Distributed Controller of Truck Platoon

A. Longitudinal Controller

The longitudinal switching function of following truck i is defined as follows

$$s_{x,i}(t) = \dot{e}_{x,i}(t) + c_1 e_{x,i}(t) + c_2 e^q_{x,i}(t) \tag{10}$$

where c_1 and c_2 are positive constant parameters; q is the ratio of two positive odd numbers and $0 < q < 1$.

Combining (1), (5) and (10), the longitudinal control law representing the desired total longitudinal force of following truck i can be obtained.

$$\begin{aligned} F_{x,i}(t) &= F_{xf,i}(t) + F_{xr,i}(t) \\ &= m\left(\sigma_1 a_{x,0}(t) + \sigma_2 a_{x,i-1}(t) - c_1 e_{v,i}(t)\right) \\ &\quad - m c_2 q e^{q-1}_{x,i}(t) e_{v,i}(t) - m k_1 s_{x,i}(t) \\ &\quad - m\eta_1 \mathrm{sgn}\left(s_{x,i}(t)\right) - m v_{y,i}(t)\omega_i(t) \end{aligned} \tag{11}$$

where k_1 and η_1 are positive constant parameters.

The desired total torque of following truck i as follows

$$T_{d,i}(t) = R F_{x,i}(t) \tag{12}$$

B. Lateral Controller

The lateral switching function of following truck i as follows

$$s_{y,i}(t) = \dot{e}_{y,i}(t) + c_3 e_{y,i}(t) + c_4 e^p_{y,i}(t) \tag{13}$$

where c_3 and c_4 are positive constant parameters; p is the ratio of two positive odd numbers and $0 < p < 1$.

Combining (1), (7) and (13), the lateral control law representing the desired lateral force of front wheel of following truck i can be obtained.

$$\begin{aligned} F_{yf,i}(t) &= P\left(-\sigma_4 \dot{\omega}_{d,i}(t) + c_3 e_{d,i}(t)\right) \\ &\quad + P\left(c_4 p e^{p-1}_{y,i}(t) e_{d,i}(t) + k_2 s_{y,i}(t)\right) \\ &\quad + P\left(\eta_2 \mathrm{sgn}\left(s_{y,i}(t)\right) + P\sigma_3 v_{x,i}(t)\omega_i(t) + Q F_{yr,i}(t)\right) \end{aligned} \tag{14}$$

where k_2 and η_2 are positive constant parameters. The expansion of the representative term in (14) is shown below

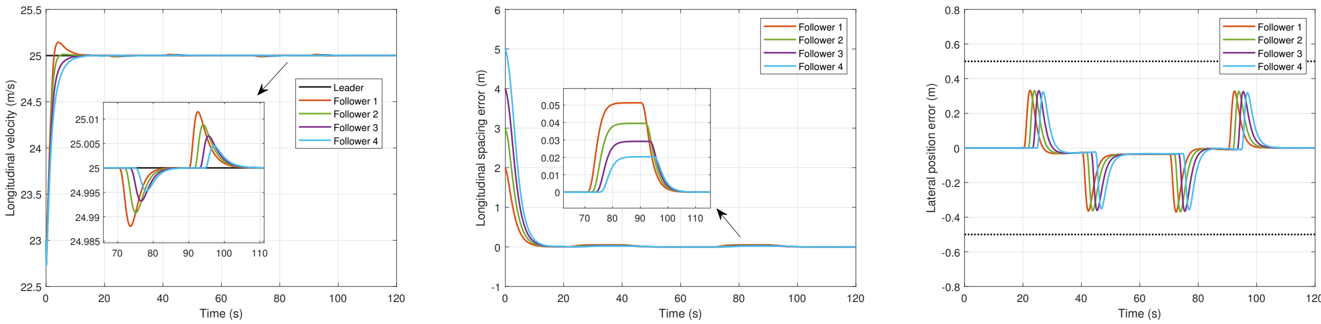
$$\begin{cases} P = -\dfrac{m I_z}{\sigma_3 I_z + m a \sigma_4} \\ Q = \dfrac{\sigma_3 + b\sigma_4}{\sigma_3 I_z + m a \sigma_4} \end{cases} \tag{15}$$

Therefore, Combining (2), (3) and (14), the desired steering angle of the front wheel of following truck i as follows

$$\delta_{f,i}(t) = \alpha_{f,i}(t) + \arctan\left(\frac{v_{y,i}(t) + a\omega_i(t)}{v_{x,i}(t)}\right) \tag{16}$$

IV. Simulation Experiment and Result Analysis

We set the lead truck to drive with the initial speed of 25 m/s at a constant speed; the initial speed of the following trucks is 23 m/s; the initial longitudinal position errors of the following trucks are $[2, 3, 4, 5]$ m, respectively; the initial lateral position errors and yaw angle errors are 0. The experimental results are shown in Fig. 1(a) to Fig. 1(c).



(a) (b) (c)

Fig. 1. (a) longitudinal velocities of truck platoon, (b) longitudinal spacing errors of following trucks, (c) lateral position errors of following trucks.

V. Conclusion

The research content of this paper is to design a distributed longitudinal and lateral controller for the following trucks, so that it can ensure longitudinal and latera motion steadily.

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